An analytical model for the fractional efficiency of a uniflow cyclone with a tangential inlet

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Abstract

An analytical model was developed to predict the fractional efficiency of a uniflow cyclone with a tangential inlet. The analysis showed that the separation efficiency is a function of particle Stokes number and the geometry of the cyclone body. Six sets of experiments were conducted under different conditions to validate the model. The experimental fractional efficiencies were determined by the total mass efficiency and the corresponding size distributions measured by using an offline particle sizer. Overall the experiments agreed with the modeling results well. Both model and experiments showed that the efficiency of this cyclone reached 99.5% and above when \(St_k>1.0\).

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1. Introduction

Cyclones have been widely used for separating airborne particles from gases in powder industry. A cyclone is relative simple to fabricate and it requires low maintenance and low operation cost. In addition, it allows continuous material removal with less clogging than a fabricate filter.

While extensive research has been focused on traditional reverse-flow cyclones, little was on the uniflow cyclones. Recent studies of the uniflow cyclones were mainly focused on vane axial ones. Vaughan [1] designed and constructed an axial flow cyclone separator with a sharp size cut-off. His study was focused on a small axial flow cyclone that is mainly for aerosol sampling. Gauthier et al. [2] tested a 5-cm diameter uniflow cyclone for the ultra-rapid fluidized reactor process at high solid loading. It was found that the cyclone collection efficiency was influenced by the separation length and the optimized length increased with the inlet air velocity. Experiments using glass beads with a mean diameter of 29 μm at high solid loading conditions showed that the optimal lengths varies from 1.0 to 10.5 times of the cyclone diameter.

Ogawa et al. [3] developed a model for the fractional efficiency of an axial flow cyclone. In the model, diffusion effect of the solid particles was considered and the decay effect of the tangential velocity of gas flow along the concave wall surface was also included. This model was validated using experimental results from a simplified axial flow cyclone with body diameter of about 10 cm. It was found that configuration influenced the performance of the cyclone, and the best performance was observed at flow gap ratio of 2.2, which was the ratio of the flow gap to the cyclone diameter.

Maynard [4] studied an axial flow cyclone under laminar flow conditions. A mathematical model was developed for low Reynolds number. In this laminar flow model, aerosol penetrations at the vane section and the rest of the cyclone body were both considered. It showed that the efficiency at the vane section vanes was dependent on several factors including the dimension and the number of vane turns. However the efficiency in the cyclone body depended on body length only. Experimental data showed a fair agreement with the model in terms of cut size.

A model for a vane axial cyclone to predict the particle separation efficiencies both under laminar and perfect mixing flow conditions was reported by Zhang [5]. The tangential air velocity, the diameters of the inner and outer tubes, the length of...
the separation chamber and the vane angle were considered in this model. This model was validated using experiments by using a uniflow cyclone with a body diameter of 25 cm.

Multi-stage axial flow cyclone separators were developed for flue gas cleaning for a fluid catalytic cracking process [6]. The three-stage cyclone separators showed that the system could handle a large volume of flue gas at a high efficiency. The corresponding cut size was 2 μm in diameter. Meanwhile, the pressure drop was much higher than that of a single stage one.

Most of the uniflow cyclones mentioned above were vane axial and small in size. It has been accepted that a vane-axial uniflow cyclone has a low pressure drop. The particle separation efficiencies, however, depend on the configuration, size, and operation conditions of the cyclones. On the other hand, the vanes, especially the complex vanes are costly to fabricate, therefore, some industries prefer a rectangular tangential inlet for its simplicity. A rectangular inlet can be fabricated in most shops.

The objective of this study is to develop a model to predict the fractional efficiency of a uniflow cyclone with tangential inlet. However, to the best knowledge of the author, there has been almost no such a model that can be used to predict the fractional efficiency of a uniflow cyclone with a tangential inlet. The closest one is the one reported by Crawford in his textbook [7]. But it was derived from a traditional reverse flow cyclone. More importantly, this model gives an implicit formula, with two factors to be determined. In order to use this model, the efficiency for particles at a certain size has to be known in advance, for example, 99% at 20 μm. Then an experiment has to be conducted first. This limits its application in predicting a performance of a cyclone. Therefore, a new analytical model for the fractional efficiency is necessary and developed in this work.

2. Theoretical

Consider a typical uniflow cyclone with a tangential inlet as shown in Fig. 1. The radiuses of the outer and inner tubes are \( r_2 \) and \( r_1 \), respectively. The height and width of the rectangular inlet are \( H \) and \( W \), respectively. The effective separation length \( L \) is from the center of the inlet \((H/2)\) to the end of the solid inner tube, because the center of the inlet represents the average start point of the traveling distance with more above and less below. Assuming no leaking and the gas is incompressible; the flow rate \( Q \) is constant from the inlet through the annular chamber and the outlet.

Since the airflow through a cyclone in an engineering application is mostly likely to be turbulent, this study is focused on a turbulent model. Consider the effect of a layer close to the wall of the outer tube, as illustrated in Fig. 2, over an infinitesimal period of time \( dr \) that corresponds to the elemental angle \( d\theta \) the particles enter this layer are assumed to be captured. Then the fraction of the particles captured during this \( dr \) in the layer is,

\[
-\frac{dN}{N} = \frac{dr}{r_2 - r_1}
\]  

(1)

During an infinitesimal period of time \( dr \), a particle travels over the angle of \( d\theta \), and meanwhile it moves outward a distance of \( \delta \). Then,

\[
dr = \frac{\delta}{V_{r_2}} = \frac{r_2 d\theta}{V_{r_2}}, \quad \text{which gives} \quad \delta = \frac{r_2 V_{r_2} d\theta}{V_{r_2}}
\]  

(2)

Put Eq. (2) into Eq. (1) and we have

\[
-\frac{dN}{N} = \frac{V_{r_2}}{V_{r_2}} \frac{r_2}{(r_2 - r_1)} d\theta
\]  

(3)

Integration using the boundary condition at the inlet \( N=N_0 \) when \( \theta=0 \) gives the number of particles remaining in the gas at the angle \( \theta \):

\[
N = N_0 \exp \left( -\frac{V_{r_2}}{V_{r_2}} \frac{r_2}{(r_2 - r_1)} \theta \right)
\]  

(4)

And it will give the efficiency at the angle \( \theta \):

\[
\eta_0 = 1 - \frac{N}{N_0} = 1 - \exp \left[ -\frac{V_{r_2}}{V_{r_2}} \frac{r_2}{(r_2 - r_1)} \theta \right]
\]  

(5)

The time for a particle traveling along a cyclone with axial distance of \( z \) at an axial velocity \( V_z \) is \( t = \frac{z}{V_z} \), which gives,

\[
\frac{r_2 \theta}{V_{r_2}} = \frac{z}{V_z} \quad \text{and} \quad \theta = \frac{V_{r_2} z}{V_z r_2}
\]  

(6)
Assuming there is no slipping between the particle and the air along axial direction, the average axial velocity in the annular tunnel is

\[ V_z = \frac{Q}{\pi (r_2^2 - r_1^2)} \]  

Approaching the outer wall at \( r_2 \), the centrifugal force on a particle is balanced by a drag on the same particle, assuming Stokes region and neglecting the slipping correction factor for micron particles,

\[ \rho_p d_p^3 V_{\theta_1}^2 / 6 r_2 = 3 \pi \mu d_p V_{r_2} \]  

It gives,

\[ \frac{V_{r_2}}{V_{\theta_1}} = \frac{\rho_p d_p^2}{18 \mu r_2} V_{r_2} \]  

Putting Eqs. (6–8) into Eq. (5) will give,

\[ \eta_0 = 1 - \exp \left[ -\left( \frac{\rho_p d_p^2 V_{\theta_1}}{18 \mu r_2} \right) \frac{r_2}{(r_2 - r_1)} \left( \frac{V_{\theta_1}}{Q} \right) \frac{z}{r_2} \right] \]

\[ = 1 - \exp \left[ -\left( \frac{\rho_p d_p^2}{18 \mu} \frac{\pi (r_2 + r_1)z}{Q r_2} V_{\theta_1} \right) \right] \]

Through a rigorous fluid mechanics analysis Crawford [7] gave \( V_{\theta_1} = V \). In this case, the average velocity at the inlet entering the body of the cyclone is,

\[ V = \frac{Q}{HW} \]

Then

\[ \eta_0 = 1 - \exp \left[ -\pi \left( \frac{\rho_p d_p^2 V}{18 \mu} \right) \frac{(r_2 + r_1)}{r_2} \frac{L}{HW} \right] \]

Replacing \( z \) with \( L \) gives the total efficiency of a cyclone for particles with a diameter of \( d_p \) as

\[ \eta = 1 - \exp \left[ -\pi \left( \frac{\rho_p d_p^2 V}{18 \mu} \right) \frac{(r_2 + r_1)}{r_2} \frac{L}{HW} \right] \]

Reorganizing the formula we can get the particle separation efficiency in terms of the Stokes number of the particle

\[ \eta = 1 - \exp \left[ -(\text{Stk}) \frac{\pi (r_2 + r_1) L_c}{r_2} \frac{L}{HW} \right] \]

where \( \text{Stk} \) is the stokes number of the particle at the inlet and

\[ \text{Stk} = \frac{\rho_p d_p^2 V}{36 \mu L_c} \]

The characteristic length at the inlet of the cyclone is defined by

\[ L_c = \frac{4A}{P} - \frac{2HW}{H + W} \]

And the corresponding Stokes number is

\[ \text{Stk} = \frac{\rho_p d_p^2 V}{36 \mu} \left( \frac{1}{H} + \frac{1}{W} \right) \]

Then the efficiency formula becomes,

\[ \eta = 1 - \exp \left[ -2\pi (\text{Stk}) \left( 1 + \frac{r_1}{r_2} \right) \left( \frac{L}{H + W} \right) \right] \]

This analytical result shows that the particle separation efficiency increases with the particle Stokes number (Stk) and the geometry of the cyclone body, including the ratio of radiiuses \( (r_1 / r_2) \) the length of the annular chamber \( (L) \), and the height \( (H) \) and width \( (W) \) of the inlet. This formula indicates that increasing the Stokes number, the ratio of inner to outer radiiuses and the length of the separation length will have a positive effect on the particle separation efficiency.

3. Experimental

A laboratory prototype of a uniflow cyclone is developed to evaluate the model introduced above. The height and width of the inlet are 7.25 in. (18.41 cm) and 1.25 in. (3.18 cm), respectively. For experiments herein, \( r_1 = 8.26 \) cm, \( r_2 = 12.07 \) cm and the effective straight section is \( L = 20 \) cm. Air flow velocity was measured by using a thermal anemometer.

Aerosol was generated using the compressed air mixing with the dust. A 0.5 m long adaptor was added in front of the rectangular inlet of the cyclone to redistribute the particles in the air. ISO 12103-1, A3 Medium test dust and sulfur powder were used in this study. The specific gravities of the A3 dust particles and the sulfur powder are 2.65 and 2.07, respectively.

Downstream the cyclone, the particles that penetrated through the cyclone were collected using a HEPA filter before air was discharging into the atmosphere. According to the specification
of the supplier, the filter can capture particles down to 0.3 μm at an efficiency of 99.99%. The total separation efficiencies by mass were determined by measuring the weights of dust fed in and the dust collected by the cyclone. A MB-600PL precision scale was used to measure the mass of the dust. Its capacity is 600 g with readability of 0.01 g. The total efficiency can be determined by the mass of dust fed in and the mass of the particles that penetrated through the cyclone, collected on the HEPA filter.

\[
\eta_M = 1 - \frac{m_o}{m} \tag{19}
\]

In addition, samples were collected from both the HEPA filter and the bunker and they were analyzed using an offline particle sizer, Horiba’s LA-300 Laser Diffraction Particle Size Distribution Analyzer. It can measure particles between 0.1 and 600 μm in geometric diameter. For particles of the same diameter (grouped in size \(i\)), the fractional efficiency can be expressed as:

\[
\eta = 1 - \frac{m_{io}}{m_i} = 1 - \frac{m_{io}}{m_{io} + m_{is}} \tag{20}
\]

If the frequency of the size group \(i\) is known for the bypass particles and those collected, the corresponding masses can be calculated,

\[
m_{io} = m_o f_{io} \tag{21}
\]

\[
m_{is} = m_s f_{is} \tag{22}
\]

where, \(f_{io}\) and \(f_{is}\) are the frequency of the size group \(i\) in the bypass particles and the collected particles. Then, the fractional efficiency equation can be rewritten as following:

\[
\eta = 1 - \frac{1}{1 + \frac{m_o}{m_{io}}} = 1 - \frac{1}{1 + \frac{m_{io}}{m_{io} f_{io}}} \tag{23}
\]

The total efficiency can be rewritten as:

\[
\eta_M = 1 - \frac{1}{\frac{m}{m_o}} = 1 - \frac{1}{1 + \frac{m}{m_o}} \tag{24}
\]

which gives

\[
\frac{m_s}{m_o} = \frac{1}{1 - \eta_M} - 1 \tag{25}
\]

Substituting it into the formula for the fractional efficiency, it then can be expressed in terms of total separation efficiency and the volume fraction for both samples of the bypass particles and the collected particles:

\[
\eta = 1 - \frac{1}{1 + \left(\frac{1}{1 - \eta_M} - 1\right)^{\frac{1}{f_{io}}}} \tag{26}
\]

### 4. Results and discussions

Six experiments under different operation conditions were conducted to validate the model. The configuration and test conditions are summarized in Table 1 below. The tests were running for 5, 10 or 25 min and the four inlet air velocities varied from 7.6 to 16.8 m/s. The corresponding dust concentrations were between 1.06 and 2.02 g/m³. During the experiments, the total mass efficiency and the size distributions of the particles penetrated through the cyclone (which gives \(f_{io}\)) and those collected (which gives \(f_{is}\)) were determined using the offline particle sizer.

Fig. 3 shows the comparison between the experiments and the modeling results. Overall, the experimental data herein agreed with the model very well at all the experimental conditions. All the modeling and experimental results showed that the particle separation efficiency of this cyclone was 99.5% or above when the corresponding particle Stokes number was about 1.0.

The difference between modeling and the experimental results are shown Fig. 4. The solid dots represent the average value from the 6 sets of experimental data and the error bars are from the corresponding standard deviation. Overall, there is a good agreement between the model and the experiments for Stokes numbers above 1–2. On the other hand, the model over-predicted the efficiencies for particles with a Stokes number below 0.2 and under estimated those greater than 0.2 for the larger ones. Several reasons contribute to this discrepancy. First of all, the analytical model was developed based on the simplification that all particles are uniformly distributed at any cross section and that only centrifugal and drag forces are...
considered. In reality larger particles tend to be pushed outward by the centrifugal forces, and consequently, it is reasonable to see the actual separation efficiencies for the larger particles are higher than the predicted values. Meanwhile, the diffusion force and re-entrainment effect due to turbulence are more significant for the smaller particles than the larger ones, and consequently lower the separation efficiencies. In addition, electrostatic forces might also affect the trajectories of the small particles, and the actual effect will depend on the polarity and the direction of the electrical field between the inner and outer tube, which was unknown during the experiments. Furthermore, the size distributions of the samples taken from the HEPA filter and the bunker might be different from the actual ones. Some small particles could penetrate through the HEPA filter, and consequently their separation efficiencies were under evaluated during the experiments.

5. Summary

An analytical model was developed for the fractional efficiency for a uniflow cyclone with a tangential inlet. This model was validated using six sets of experiments that were conducted using two types of dust at different operation conditions. Overall the experiments agree with the modeling results well. Both model and experiments showed the efficiency of this cyclone reached 99.5% and above when $Stk > 1$.

Nomenclature

- $d_p$: Particle diameter (m)
- $f_{io}$: Volume fraction of the $i$th group particles penetrated through the cyclone
- $f_{is}$: Volume fraction of the $i$th group particles separated by the cyclone
- $g$: Gravitational acceleration (m/s$^2$)
- $H$: Cyclone inlet height (m)
- $L$: Effective separation length of the cyclone (m)
- $L_c$: Characteristic length of the Stokes number (m)
- $m$: Total particle mass concentration at the inlet of the cyclone (kg/m$^3$)
- $m_o$: Total mass concentration of particles penetrated through the cyclone (kg/m$^3$)
- $m_{io}$: Mass concentration of $i$th group particles penetrated through the cyclone (kg/m$^3$)
- $m_{is}$: Mass concentration of $i$th group particles separated by the cyclone (kg/m$^3$)
- $N$: The number concentration of particles at angle $\theta$ (kg/m$^3$)
- $N_0$: The initial total particle number at angle $\theta = 0$ (kg/m$^3$)
- $Q$: Volumetric gas flow (m$^3$/s)
- $r_1$: Inner radius of the annular channel (m)
- $r_2$: Outer radius of the annular channel (m)
- $Stk$: Particle Stokes number
- $V$: Average air velocity at the cyclone inlet (m/s)
- $V_{t1}$: Particle tangential velocity close to the wall (m/s)
- $V_{r2}$: Particle radial velocity close to the outer wall (m/s)
- $W$: Width of the inlet of the cyclone, (m)
- $\eta$: Fractional efficiency
- $\eta_\theta$: Collection efficiency while the particle travelled $\theta$ degree
- $\eta_M$: Total collection efficiency based on mass
- $\mu$: Air viscosity (kg m/s)
- $\rho_p$: Density of the particles (kg/m$^3$)
- $\theta$: The traveling angle of the particles

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