Chapter 11
Air Dispersion

Air Dispersion from Point Sources

- Pollutants emitted from a source are diluted by the ambient air
- Dilution reduces impact on local air quality

Air Quality Models are

- Mathematical formulations that include parameters that affect pollutant concentrations.
- Tools to
  - evaluate compliance with regulatory requirements
  - predict the performance of a design
  - Determine extent of emission reductions required
  - Evaluate sources in permit applications
- Tools for emergency planning

Air Quality Models

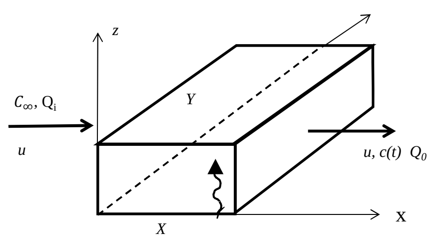
- **Air Dispersion Model**
  - Describes transport, dispersion, vertical mixing and moisture in time and space
- **Chemical Model**
  - Describes transformation of directly emitted particles and gases to secondary particles and gases;
  - also estimates the equilibrium between gas and particles for volatile species
- **Source Dispersion Model**
  - Uses the outputs from the previous models to estimate concentrations measured at receptors; includes mathematical simulations of transport, dispersion, vertical mixing, deposition and chemical models to represent transformation.
- **Receptor Model**
  - Infers contributions from different primary source emissions or precursors from multivariate measurements taken at one ore more receptor sites.

Air Dispersion Models

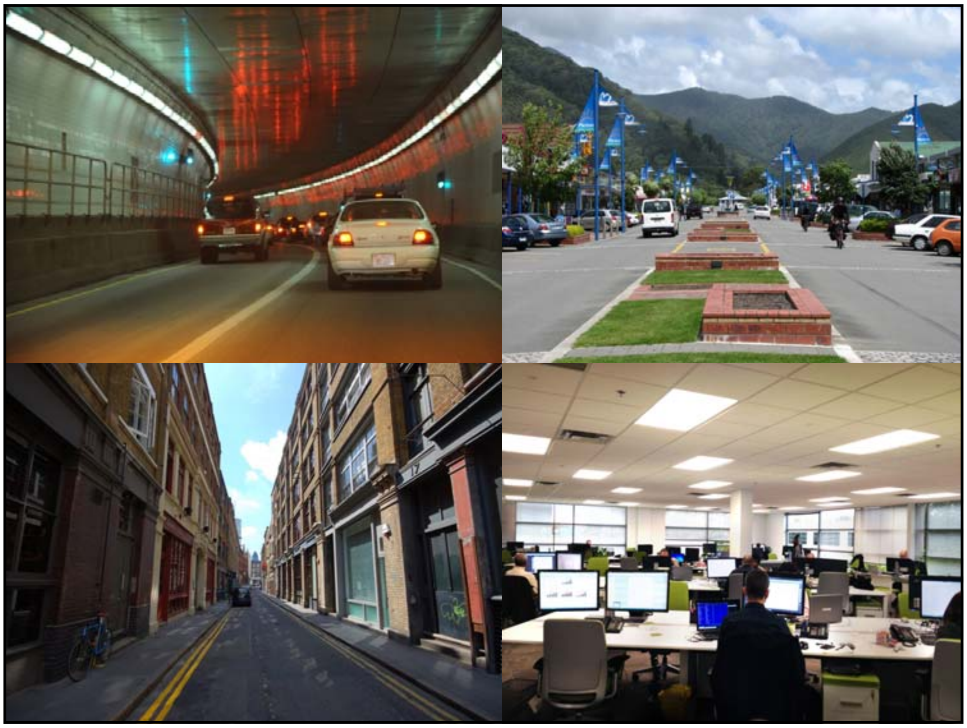
- Box Model
- Gaussian Dispersion Model
- Puff Model

The Box Model

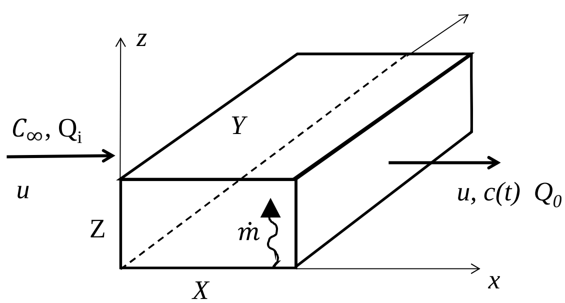
- The simplest model for dispersion at a ground level
- Box model assumptions
  - ❑ **Perfect mixing within the box (calculation domain)**
  - ❑ **Air velocity is perpendicular to the inlet surface**



c_a= *concentration of the pollutant in the ambient*
u=*velocity of air (with pollutant)*
c = *concentration of the pollutant within the box and the exit*
$\dot{m}$=*source term*



Box Model Equations



- Conservation of mass of the air

$$Q_i = Q_o = Q = uZY$$

- Conservation of mass of pollutant

$$ZYX\frac{dC}{dt} = (\dot{m} + uZYC_i) - uZYC$$

$$\frac{dC}{(\dot{m} + uZYC_i) - uZYC}$$

$$= \frac{1}{\quad} dt$$

Model Solution (U, $\dot{m}$, C_a are constants)

$$\int_{C\infty}^{C} \frac{dC}{(\dot{m} + uz_x Y C_i) - uhwC} = \frac{1}{z_x Y X} t$$

- When u, $\dot{m}$, C_i are constants, the steady-state concentration C_{ss} in the box can be determined from $dC/dt = 0$.

$$C_{ss} = C_i + \frac{\dot{m}}{uZY}$$

- Integrating Equation (11-3) from time zero to any time leads to

$$\frac{C(t) - C_i}{C_{ss} - C_i} = 1 - \exp\left(-\frac{ut}{X}\right)$$

X is the length of the box in the direction of the airflow

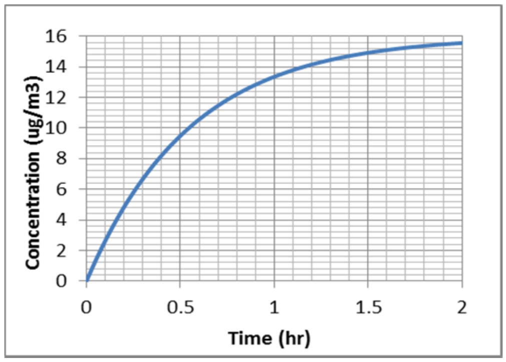
Source strength and inlet air speed may NOT be constant

Example 11.1: Air Pollution in City Block

- A city street is Y = 25 m wide with high rise buildings of 100 m high, which traps pollutants below 100 m. In a rush hour, cars line up on the X=1 km long street and emit air pollutants continuously. Assume steady emission rate, wind speed and direction. The particulate air pollutant emission rate is 20 µg/s per meter of street and constant wind blowing at a steady speed of u = 0.5 m/s. Plot the concentration of the air pollutant in the street against time.

Solution

- $\dot{m} = 20\,\mu g/s.m \times 1000\,m = 20{,}000\,\mu g/s$
- $C_{ss} = \frac{\dot{m}}{uYZ} = \frac{20{,}000}{0.5\times25\times100} = 16\,\mu g/m^3$
- $C(t) = C_{ss}\left[1 - \exp\left(-\frac{ut}{X}\right)\right] = 16\left[1 - \exp\left(-\frac{0.5\times t}{1000}\right)\right]$



Gaussian Dispersion Model

- Gaussian models are the most widely used for estimating the impact of nonreactive air pollutants.
- A Gaussian-plume model can be used to predict the downwind concentration resulting from this point source under a specific atmospheric condition.
- It is a material balance model for a point source such as a power plant stack.

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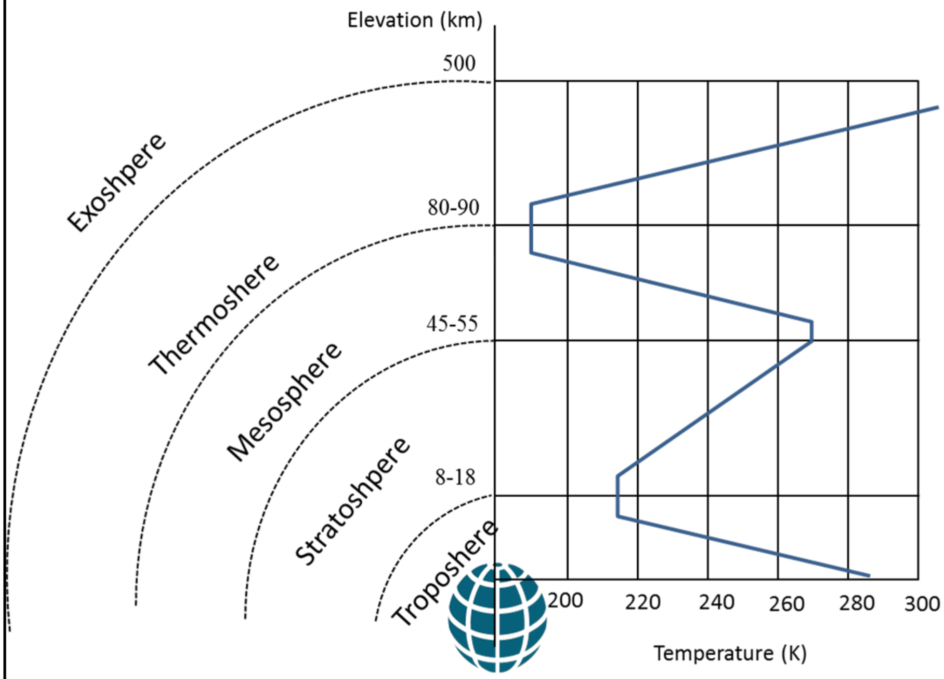

- Air dispersion takes place primarily in the lower layers of the atmosphere which interacts with the surface of the earth.
- Sometimes referred to as ground boundary layer, the planetary boundary layer (PBL) is the lowest layer of the troposphere where wind is influenced by friction.
- At night and in the cool season the PBL tends to be lower in thickness
- during the day and in the warm season it tends to have a higher thickness.

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Atmospheric Air Density and Pressure

$$\rho = \frac{PM}{RT}$$

- Notice that M changes with the water content
- Increased amount of water content (increase/decrease?) the value of M (air).

$$\rho = -\frac{1}{g}\frac{dP}{dz}$$

$$\frac{dP}{dz} = -\frac{gMP}{RT}$$

- *If M and T are constant*

$$P(z) = P_0 exp\left(-\frac{gM}{RT}z\right)$$

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Air Turbulence

- Circular eddies of air movements over short timescales than those that determine wind speed (unstable)
- **Mechanical Turbulence**:
  - Caused by air moving over and around structures/vegetation
  - Increases with wind speed
  - Affected by surface roughness
- **Thermal Turbulence:**
  - Caused by heating/cooling of the earth's surface
  - Flows are typically vertical
  - Convection cells of upwards of 1000 - 1500 meters

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Air Parcel

- An imaginary body of air to which may be assigned any or all of the basic dynamic and thermodynamic properties of atmospheric air.
- In air dispersion, it is most likely part of the “air” from the emission source
- A parcel is large enough to contain a very great number of molecules, but small enough so that the properties assigned to it are approximately uniform within it and so that its motions with respect to the surrounding atmosphere do not induce marked compensatory movements.
- It cannot be given precise numerical definition, but a cubic foot of air might fit well into most contexts where air parcels are discussed, particularly those related to static stability.

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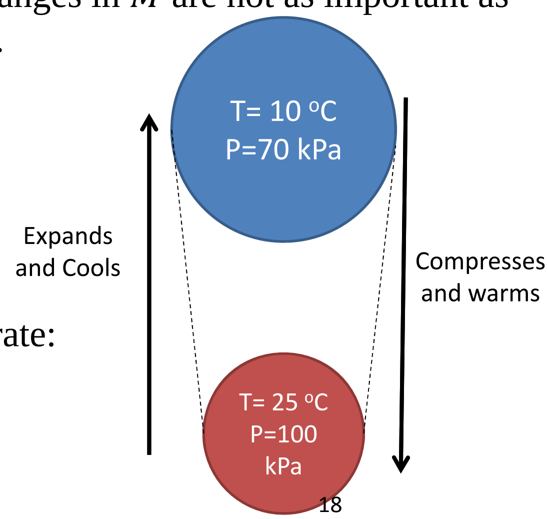
Adiabatic Lapse Rate of Temperature

- Both temperature T and molar weight M change with elevation, and the changes in M are not as important as those of temperature.

$$\frac{dT}{dz} = -\frac{g}{c_{p,a} + h_{fg}\left(\frac{dw}{dT}\right)}$$

- Dry-adiabatic lapse rate: 9.85 °C/km



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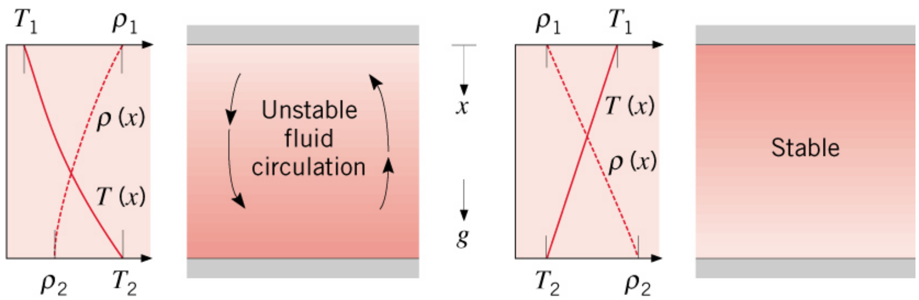
Stable and Unstable Temperature Gradients



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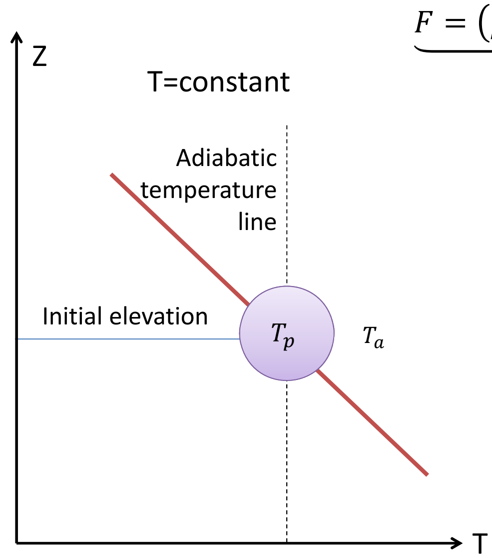
Atmosphere Stability



$$\underbrace{F = (\rho_a - \rho_p)Vg}= \rho_p V \frac{dv}{dt}$$

$$\rightarrow \frac{dv}{dt} = \left(\frac{\rho_a}{\rho_p} - 1\right) g$$

$$\rho = \frac{MP}{RT}$$

$$\frac{dv}{dt} = \left(\frac{T_p}{T_a} - 1\right) g$$

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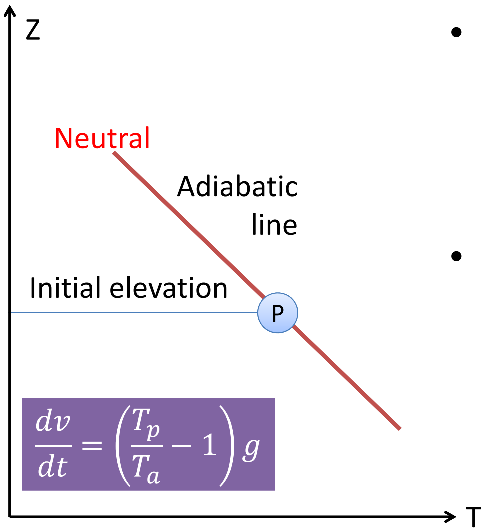
Neutral atmosphere



$$\frac{dv}{dt} = \left(\frac{T_p}{T_a} - 1\right) g$$

- When the air parcel temperature elapse along elevation is the same as that of the surrounding air
- the air parcel will stay still with respect to the surrounding air.

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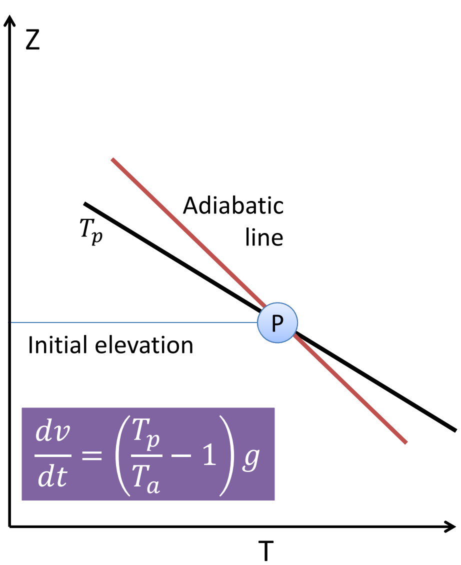
Stable atmosphere



$$\frac{dv}{dt} = \left(\frac{T_p}{T_a} - 1\right) g$$

- When the air parcel temperature elapse along elevation is greater than that of the surrounding atmospheric air,
- the air parcel is colder than the surrounding air when it moves up, or hotter when it goes down.
- As a result, the surrounding air exerts a total force to move the air parcel back to its original position.

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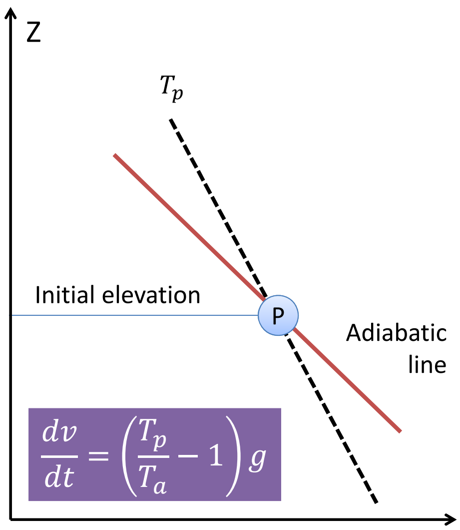
Unstable atmosphere



- When the air parcel temperature elapse along elevation is weaker than that of the surrounding air,
- the air parcel is colder than the surrounding air when it moves down Or hotter when it moves up.
- As a result, the surrounding air exerts a total force to drive the air parcel away from its original position and convection is produced.

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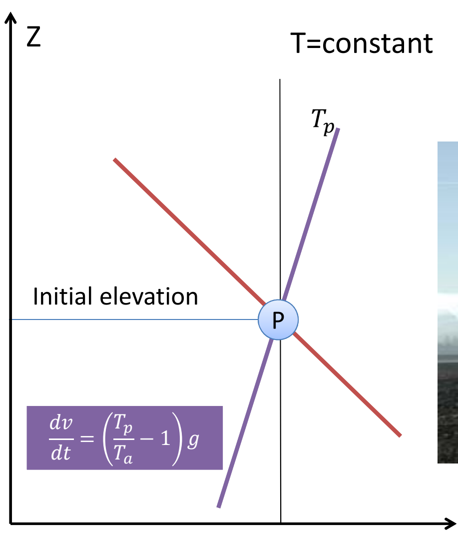
Inversion:

- when atm temperature increases with elevation (T_p<T_a); $\frac{dv}{dt} < 0$



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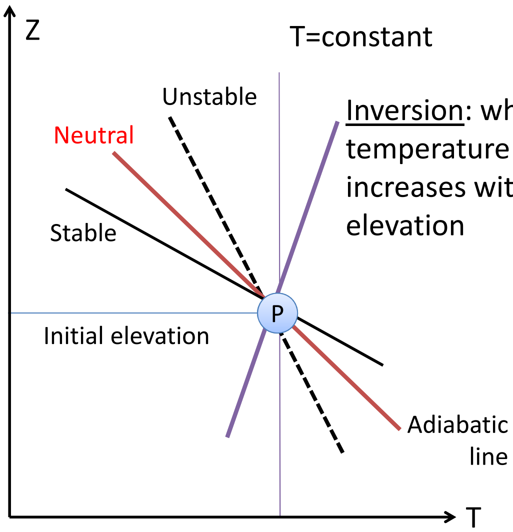
Air Parcel Stability



$$F = \underbrace{(\rho_a - \rho_p)Vg} = \rho_p V \frac{dv}{dt}$$

$$\rightarrow \frac{dv}{dt} = \left(\frac{\rho_a}{\rho_p} - 1\right) g$$

$$\rho = \frac{MP}{RT}$$

$$\frac{dv}{dt} = \left(\frac{T_p}{T_a} - 1\right) g$$

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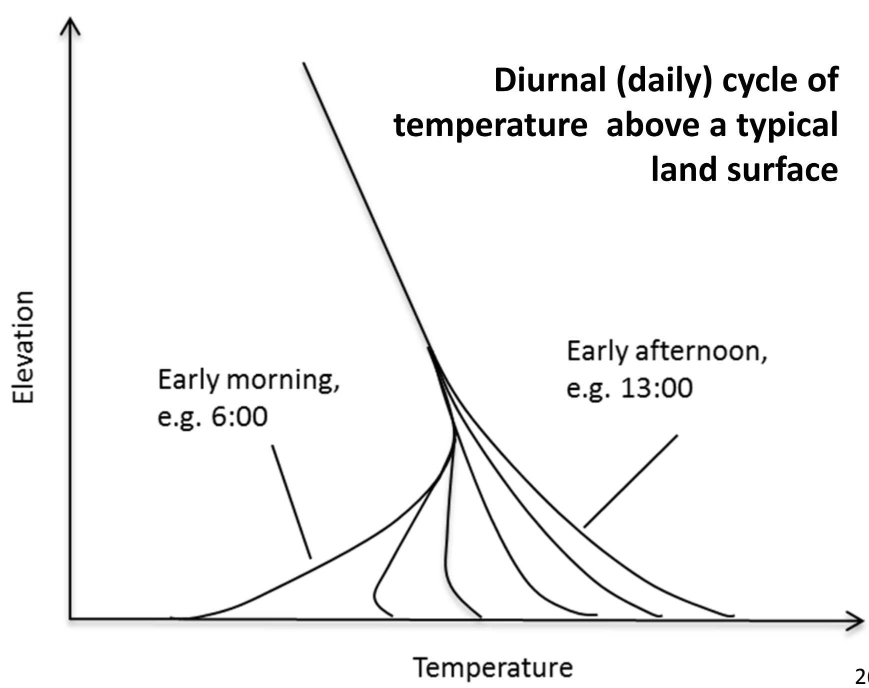
Diurnal (daily) cycle of temperature above a typical land surface



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Pasquill Stability Classes

P class	A	B	C	D	E	F
Stability	Extremely unstable	Moderately unstable	Slightly unstable	Neutral	Slightly stable	Stable

Surface wind speed (at 10 m) m/s	Day			Night	
	Incoming solar radiation			Thinly overcast or $> {}^{3}/_{8}$ cloud	Clear or $\leq {}^{3}/_{8}$ cloud
	Strong	Moderate	Slight		
0-2	A	A-B	B	-	-
2-3	A-B	B	C	E	F
3-5	B	B-C	C	D	E
5-6	C	C-D	D	D	D
>6	C	D	D	D	D

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Wind Speed

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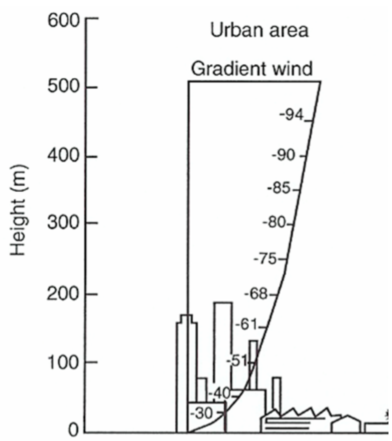
Wind Speed Profile in Neutral Atmosphere

$$\frac{u}{u_*} = \frac{1}{k} ln\left(\frac{z}{z_0}\right)$$



- z_0 is the surface roughness height, where u(@ z0)=0
- u_* is determined by measuring u at 10 m height

$$\frac{u_*}{k} = \frac{u_{10}}{ln(z_{10}/z_0)}$$

- k is the Karman constant. Although there are many values for the Karman constant and Trinh (2010) summarized all the Karman constants in literature. The most widely used value is

$$k = 0.4$$

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Terrain	Description	z_0 (m)
Urban	Roughly open with occasional obstacles	0.1
	Rough area with scatter obstacles	0.25
	Very rough areas with low buildings or industrial tanks as obstacles	0.5
	Skimming areas with buildings of similar height	1
	Chaotic city center with buildings of different heights	2
Rural	Agricultural land	0.25
	Range land	0.05
	Forrest land, wet land, forest wet land	1
	Water body	0.001
	Perennial snow or ice	0.20
Mountains	Rocky mountains	50-70
	Mountains	5-70
Ocean	$z_0 = 2 \times 10^{-6} u_{10}^{2.5}$	

Wind speed profile in Nonneutral atmosphere

$$\frac{u}{u_*} = \frac{1}{k} ln\left(\frac{z}{z_0}\right) + \frac{5}{k} ln\left(\frac{z - z_0}{L}\right)$$

Obukhov Length, L

$$\frac{1}{L} = a + b \times \log_{10}(z_0)$$

Like surface roughness height, this Obukhov length is not a physical length either.

P class	A	B	C	D	E	F
Stability	Extremely unstable	Moderately unstable	Slightly unstable	Neutral	Slightly stable	Moderately stable
a	-0.096	-0.037	-0.002	0	0.004	0.035
b	0.029		0.018	0	-0.018	-0.036

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Wind speed profile in unstable atmosphere*

- $\frac{u}{u_*} = \frac{1}{k} ln\left(\frac{z}{z_0}\right) + \underbrace{\frac{1}{k}\left\{\ln\left[\frac{(\beta_0^2 + 1)(\beta_0 + 1)^2}{(\beta^2 + 1)(\beta + 1)^2}\right] + 2[arctan(\beta) - arctan(\beta_0)]\right\}}$

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Example 11.2 Wind speed profile

- *In a rural area, the friction height is* $z_0 = 0.25\ m$*, and the wind speed measured at 10 m height is 4* m/s *under neutral condition. Plot the vertical wind speed profile.*

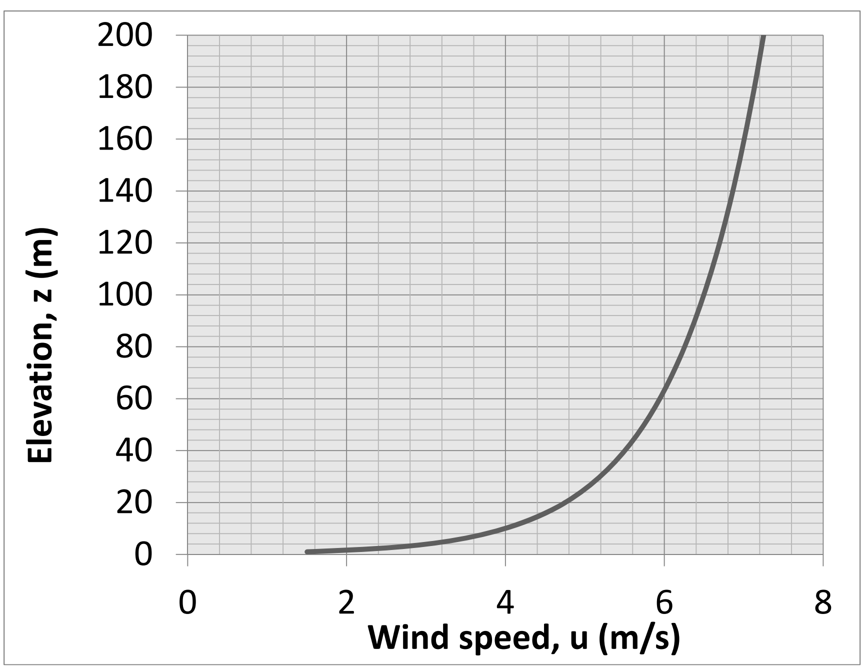
Solution:

- *Equation (11-17) gives*

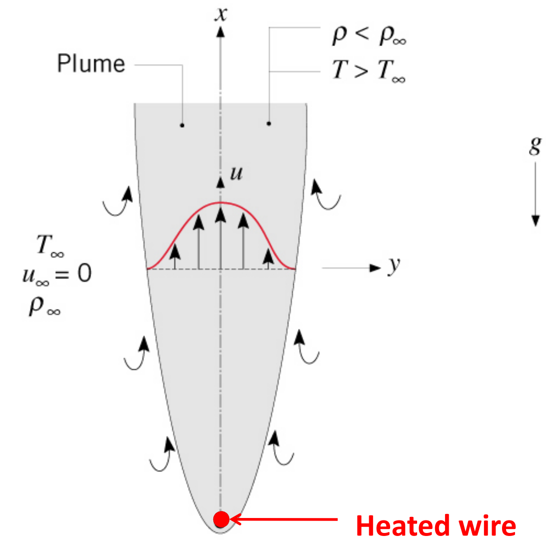
$$\frac{u_*}{k} = \frac{u_{10}}{ln(z_{10}/z_0)} = \frac{4}{ln(10/0.25)} = 1.084$$

- *Then we have the velocity as a function of elevation:*

$$u = \frac{u_*}{k} ln\left(\frac{z}{z_0}\right) = 1.084 ln(4z)$$



- ***Recall: Free convection Plume***
  - Free convection is important to air dispersion



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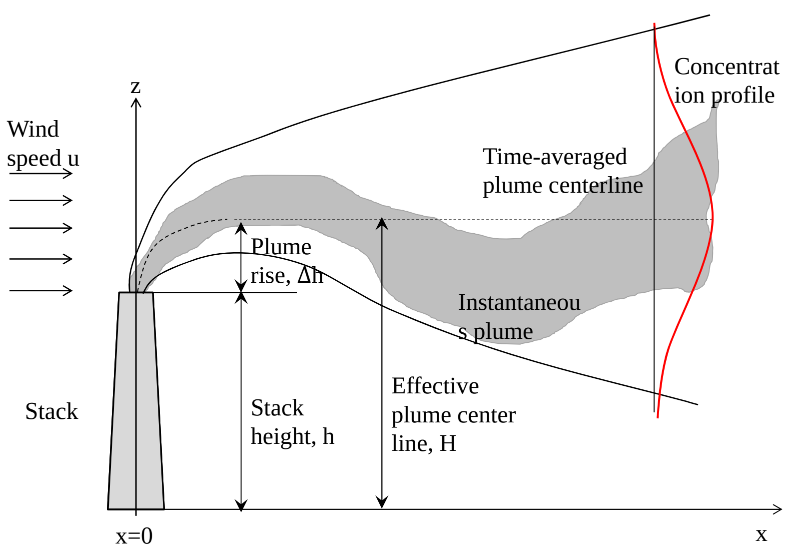
Consider a simple case (in a wind tunnel)



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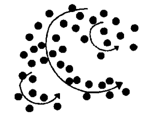
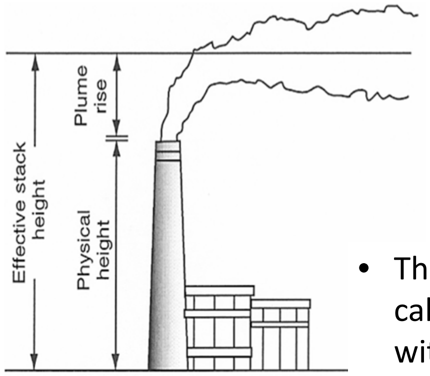
Gaussian Plume Model





- The cause of the spread of plumes is the large-scale turbulent mixing that exists in the atmosphere

- The Gaussian plume approach tries to calculate only that average value without making any statement about instantaneous values.

- The results obtained by Gaussian plume calculations should be considered only as averages over periods of at least 20 minutes, and preferably 0.5-1 hour.

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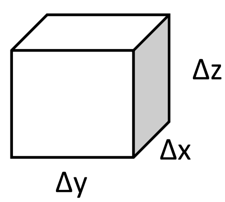
Gaussian Plume Model Derivation

- Consider a cubic control volume $\Delta x \Delta y \Delta z$ along the center of the plume ($z = H$).
- Then the net mass flow rate (ΔJ_x kg/s) along x direction is described as the differences through surfaces x and $x + \Delta x$



$$\Delta J_x = (C_x - C_{x+\Delta x}) u \Delta y \Delta z + \left[\left(-D_x \frac{\partial C}{\partial x} \right)_x - \left(-D_x \frac{\partial C}{\partial x} \right)_{x+\Delta x} \right] \Delta y \Delta z$$

$$\Delta J_y = \left[\left(-D_y \frac{\partial C}{\partial y} \right)_y - \left(-D_y \frac{\partial C}{\partial y} \right)_{y+\Delta y} \right] \Delta x \Delta z$$

$$\Delta J_z = \left[\left(-D_z \frac{\partial C}{\partial z} \right)_z - \left(-D_z \frac{\partial C}{\partial z} \right)_{z+\Delta z} \right] \Delta y \Delta x$$

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- The mass balance leads to increment of pollutant in the cubic volume over a small period of time as

$$\Delta C(\Delta x \Delta y \Delta z) = \left(\Delta J_x + \Delta J_y + \Delta J_x\right)\Delta t$$

$$\frac{\Delta C}{\Delta t} = \frac{(C_x - C_{x+\Delta x})u + \left[\left(-D_x \frac{\partial C}{\partial x}\right)_x - \left(-D_z \frac{\partial C}{\partial z}\right)_{x+\Delta x}\right]}{\Delta x} + \frac{\left[\left(-D_y \frac{\partial C}{\partial y}\right)_y - \left(-D_y \frac{\partial C}{\partial y}\right)_{y+\Delta y}\right]}{\Delta y} + \frac{\left[\left(-D_z \frac{\partial C}{\partial z}\right)_z - \left(-D_z \frac{\partial C}{\partial z}\right)_{z+\Delta z}\right]}{\Delta z}$$

$$\Rightarrow \frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial x} + D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2}$$

$$\frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial x} + D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2}$$

Assumptions:

1) Steady state: $\frac{\partial C}{\partial t} = 0$

2) x-direction transport by wind is much greater than that by eddy diffusion

$$u\frac{\partial C}{\partial x} \gg D_x \frac{\partial^2 C}{\partial x^2}$$

Boundary Conditions

$$C = 0 \text{ as } x, y, z \to \infty$$

$$C \to \infty \text{ at } x, y, z \to 0$$

$$D_z \frac{\partial C}{\partial z} = 0 \text{ at } z \to 0 \text{ and } x, y > 0 \text{ (wall boundary)}$$

$$\int_{-\infty}^{\infty}\int_{0}^{\infty} uC(y,z)\,dz\,dy = \dot{m} \text{ at x} > 0 \quad \text{(conservation of mass)}$$

- we can get the air pollutant concentration at any point (x,y,z) as

$$C(x,y,z) = \frac{\dot{m}}{2\pi x\sqrt{D_y D_z}} \exp\left[\left(-\frac{u}{4x}\right)\left(\frac{y^2}{D_y} + \frac{z^2}{D_z}\right)\right]$$

- Define σ_y and σ_z are the dispersion coefficients in the transverse (y)and vertical (z) direction, respectively

$$\sigma_y^2 = \frac{2xD_y}{u} \text{ and } \sigma_z^2 = \frac{2xD_z}{u}$$

$$C(x,y,z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z}\frac{1}{u}\exp\left[-\frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2}\right]$$

$$C(x,y,z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z}\frac{1}{u}\exp\left[-\frac{y^2}{2\sigma_y^2} - \frac{(z-H)^2}{2\sigma_z^2}\right]$$

Stability class	Open/Rural Sites		Urban/Industrial Sites	
	$\sigma_y(m)$	$\sigma_z\,(m)$	$\sigma_y(m)$	$\sigma_z\,(m)$
A	$\frac{0.22x}{\sqrt{1+0.0001x}}$	$0.20x$	$\frac{0.32x}{\sqrt{1+0.0004x}}$	$0.24x\sqrt{1+0.001x}$
B	$\frac{0.16x}{\sqrt{1+0.0001x}}$	$0.12x$		
C	$\frac{0.11x}{\sqrt{1+0.0001x}}$	$\frac{0.08x}{\sqrt{1+0.0002x}}$	$\frac{0.22x}{\sqrt{1+0.0004x}}$	$0.20x$
D	$\frac{0.08x}{\sqrt{1+0.0001x}}$	$\frac{0.06x}{\sqrt{1+0.0015x}}$	$\frac{0.16x}{\sqrt{1+0.0004x}}$	$\frac{0.14x}{\sqrt{1+0.0003x}}$
E	$\frac{0.06x}{\sqrt{1+0.0001x}}$	$\frac{0.03x}{1+0.0003x}$	$\frac{0.11x}{\sqrt{1+0.0004x}}$	$\frac{0.08x}{\sqrt{1+0.0015x}}$
F	$\frac{0.04x}{\sqrt{1+0.0001x}}$	$\frac{0.016x}{1+0.0003x}$		

Gaussian Dispersion Model Equation

$$C(x, y, z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z}\frac{1}{u}\exp\left[-\frac{y^2}{2\sigma_y^2} - \frac{(z-H)^2}{2\sigma_z^2}\right]$$

- where σ_y and σ_z are the dispersion coefficients in the transverse (y)and vertical (z) direction, respectively.
- Different values of σ_y & σ_z mean that spreading in the vertical and horizontal direction are not equal.
- Most often $\sigma_y > \sigma_z$, so that at a given x a contour of constant concentration is like an ellipse, with the long axis horizontal.

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Comments

$$C(x, y, z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z}\frac{1}{u}\exp\left[-\frac{y^2}{2\sigma_y^2}\right]\exp\left[-\frac{(z-H)^2}{2\sigma_z^2}\right]$$

- If we set y = (z – H) = 0, then the two right-most terms will be exp(0) = 1, which shows that the first term is the concentration on the centerline of the plume.

$$C(x, y = 0, z = H) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z}\frac{1}{u}$$

- The values of σ_y & σ_z increase with downwind distance, so that this centerline concentration decreases with downwind distance.

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More comments on

$$C(x,y,z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z}\frac{1}{u}\exp\left[-\frac{y^2}{2\sigma_y^2}\right]\exp\left[-\frac{(z-H)^2}{2\sigma_z^2}\right]$$

- The second term shows how the concentration decreases as the plume move in the horizontal sidewise, y, direction from the plume centerline.
- Because the second term involves y^2 it is the same for moving in the + or –y direction. The third term is like the second, but it shows how the concentration decreases as we move vertically away from the elevation of the plume centerline (z = H). It also is symmetrical.

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$$C(x,y,z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z}\frac{1}{u}\exp\left[-\frac{y^2}{2\sigma_y^2}-\frac{(z-H)^2}{2\sigma_z^2}\right]$$

σ_y and σ_z

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Stability class	Open/Rural Sites		Urban/Industrial Sites	
	$\sigma_y(m)$	$\sigma_z\ (m)$	$\sigma_y(m)$	$\sigma_z\ (m)$
A	$\frac{0.22x}{\sqrt{1+0.0001x}}$	$0.20x$	$\frac{0.32x}{\sqrt{1+0.0004x}}$	$0.24x\sqrt{1+0.001x}$
B	$\frac{0.16x}{\sqrt{1+0.0001x}}$	$0.12x$		
C	$\frac{0.11x}{\sqrt{1+0.0001x}}$	$\frac{0.08x}{\sqrt{1+0.0002x}}$	$\frac{0.22x}{\sqrt{1+0.0004x}}$	$0.20x$
D	$\frac{0.08x}{\sqrt{1+0.0001x}}$	$\frac{0.06x}{\sqrt{1+0.0015x}}$	$\frac{0.16x}{\sqrt{1+0.0004x}}$	$\frac{0.14x}{\sqrt{1+0.0003x}}$
E	$\frac{0.06x}{\sqrt{1+0.0001x}}$	$\frac{0.03x}{1+0.0003x}$	$\frac{0.11x}{\sqrt{1+0.0004x}}$	$\frac{0.08x}{\sqrt{1+0.0015x}}$
F	$\frac{0.04x}{\sqrt{1+0.0001x}}$	$\frac{0.016x}{1+0.0003x}$		

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Example 11.3 Gaussian Plume Model

- *In a bright sunny day, the wind speed is assumed to be 6 m/s and horizontal. A power plant in a rural area with a stack of 100 m high continuously discharge SO_2 into the atmosphere at a stable rate of 0.1 kg/s. The plume rise is 20 m. Ignoring the chemical reactions in the atmosphere,*

$$\sigma_y = \frac{0.11x}{\sqrt{1+0.0001x}};\ \sigma_z = \frac{0.08x}{\sqrt{1+0.0002x}}$$

a) *Estimate the SO_2 concentration at the center of the plume 5 km downwind from the stack.*
b) *Estimate the ground level SO_2 concentration 5 km downwind*
c) *Plot the ground level concentration right under the plume*

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Solution

$$C(x,y,z)=\frac{\dot{m}}{2\pi\sigma_y\sigma_z}\frac{1}{u}\exp\left[-\frac{y^2}{2\sigma_y^2}-\frac{(z-H)^2}{2\sigma_z^2}\right]$$

$$\sigma_y=\frac{0.11x}{\sqrt{1+0.0001x}}=\frac{0.11\times 5000}{\sqrt{1+0.0001\times 5000}}=449.1\ m$$

$$\sigma_z=\frac{0.08x}{\sqrt{1+0.0002x}}=\frac{0.08\times 5000}{\sqrt{1+0.0002\times 5000}}=282.8\ m$$

a) The concentration at the plume center 5 km downwind is calculated using Equation (above) *with z-H =0 and y=0*

$$C(x,0,H)=\frac{\dot{m}}{2\pi\sigma_y\sigma_z u}$$

$$=\frac{0.1}{2\pi\times 449.1\times 282.8\times 6}=2.09\times 10^{-8}\ (kg/m^3)$$

$$=20.9\ (\mu g/m^3)$$

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$$C(x,y,z)=\frac{\dot{m}}{2\pi\sigma_y\sigma_z}\frac{1}{u}\exp\left[-\frac{y^2}{2\sigma_y^2}-\frac{(z-H)^2}{2\sigma_z^2}\right]$$

b) The concentration at the ground right below the plume center 5 km downwind is calculated using Equation (above) *with z=0, y=0 and H=120:*

$$C(x,y,z)=\frac{\dot{m}}{2\pi\sigma_y\sigma_z u}\exp\left[-\frac{H^2}{2\sigma_z^2}\right]$$

$$=20.9\times\exp\left[-\frac{1}{2}\left(\frac{120}{282.8}\right)^2\right]=20.9\times 0.914$$

$$=19.1(\mu g/m^3)$$

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$$C(x,y,z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z}\frac{1}{u}\exp\left[-\frac{y^2}{2\sigma_y^2} - \frac{(z-H)^2}{2\sigma_z^2}\right]$$

c) The concentration at the ground right below the plume center 5 km downwind is calculated using Equation (above) *with z=0, y=0 and H=120:*

$$C(x,y,z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z u}\exp\left[-\frac{H^2}{2\sigma_z^2}\right]$$

$$= \frac{0.1}{2\pi\frac{0.11x}{\sqrt{1+0.0001x}}\frac{0.08x}{\sqrt{1+0.0002x}}6}\exp\left[-\frac{120^2}{2\left(\frac{0.08x}{\sqrt{1+0.0002x}}\right)^2}\right]$$

$$= \frac{\sqrt{(1+0.0001x)(1+0.0002x)}}{0.331752x^2}\exp\left[-\left(\frac{1125000+225x}{x^2}\right)\right]$$

The plot is shown in Figure 11-7.

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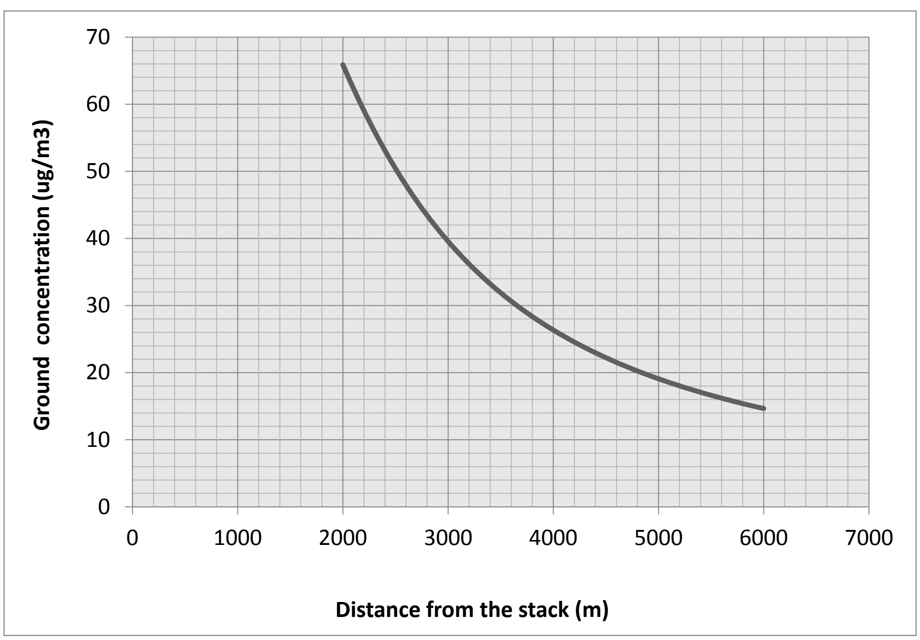

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Plume Rise

Briggs Plume Rise Formulas

- Plume rise is caused by buoyancy and momentum
- Most plumes are buoyant plumes
- Buoyancy flux parameter

$$F_B = \left(1 - \frac{\rho_s}{\rho_a}\right)\frac{d_s^2}{4} g v_s$$

$$\frac{\rho_s}{\rho_a} = \frac{M_s}{M_a}\frac{T_a}{T_s} \approx \frac{T_a}{T_s}$$

$$F_B = \left(1 - \frac{T_a}{T_s}\right)\frac{d_s^2}{4} g v_s$$

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Buoyant Plume Rise

- Transitional Plume Rise

$$\Delta h = \left(\frac{25}{6}\frac{F_B}{\bar{u}^3}x^2\right)^{1/3}$$

- Critical distance corresponding to maximum plume rise

$$x_c = \begin{cases} 49F_B^{5/8} & \text{for } F_B < 55\ \mathrm{m^4/s^3} \\ 119F_B^{2/5} & \text{for } F_B > 55\ \mathrm{m^4/s^3} \end{cases}$$

- Maximum Plume rise

$$\Delta h_m \cong \begin{cases} 21.4\dfrac{F_B^{3/4}}{\bar{u}} & \text{for } F_B < 55\ \mathrm{m^4/s^3} \\ 38.7\dfrac{F_B^{3/5}}{\bar{u}} & \text{for } F_B > 55\ \mathrm{m^4/s^3} \end{cases}$$

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Example 11.4: *Plume rise*

- *Consider a power plant stack with a diameter of d_s =1.2 m and the stack emission gas is discharged at a speed of $v_s = 5\ m/s$. Assume wind speed u=1.1 m/s, and surrounding air T_a =300 K. Plot the plume rise downwind the emission source for discharge temperature of T_s =500 K.*

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Solution

- *Since the plume from a power plant is buoyancy dominant plume, we only consider the buoyancy flux*

$$F_B = \left(1 - \frac{T_a}{T_s}\right)\frac{d_s^2}{4} g v_s$$

$$= \left(1 - \frac{300}{500}\right)\left(\frac{1.2}{2}\right)^2 9.81 \times 5 = \frac{7.063\ m^4}{s^3}$$

- *Since* $F_B < 55\ m^4/s^3$ *the corresponding maximum plume rise is calculated using*

$$\Delta h_m = \frac{21.4}{\bar{u}} F_B^{3/4} = 84.3\ m$$

- Transitional Plume Rise

$$\Delta h = \left(\frac{25}{6}\frac{F_B}{\bar{u}^3} x^2\right)^{1/3}$$

- Maximum at 84.3 m

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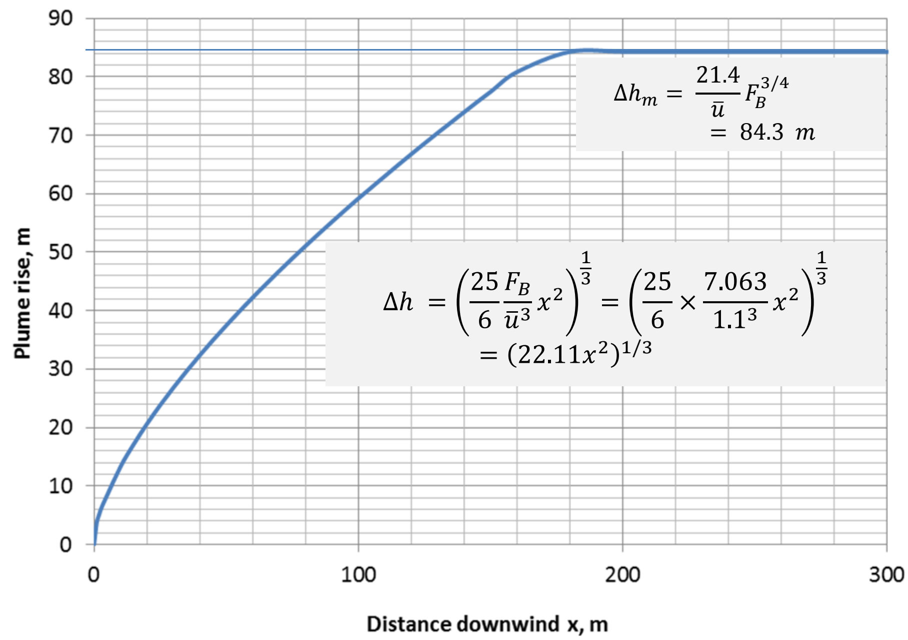

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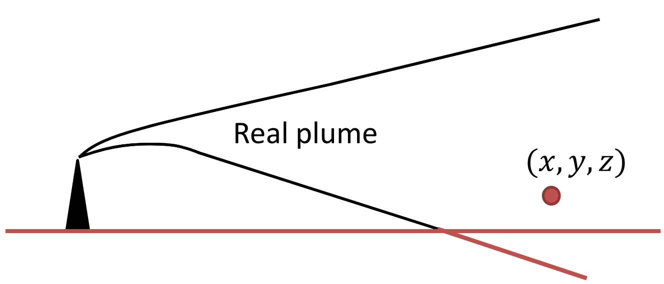
Ground surface reflection



$$C(x,y,z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z u}\exp\left[-\frac{y^2}{2\sigma_y^2}\right]\underline{\exp\left[-\frac{(z-H)^2}{2\sigma_z^2}\right]}$$

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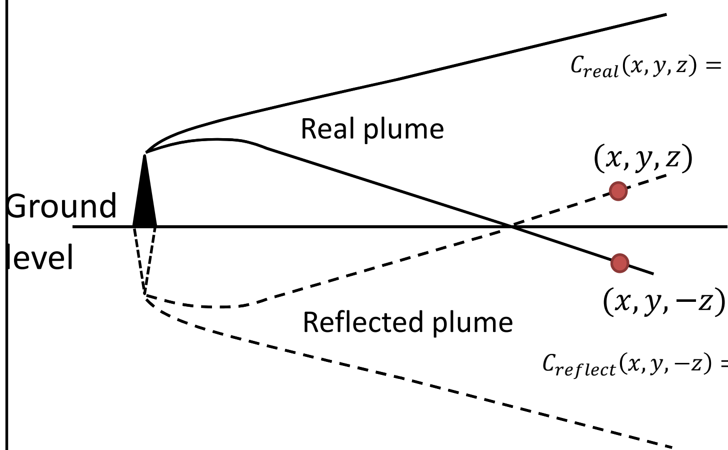
Ground surface reflection



$$C_{real}(x,y,z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z u}\exp\left[-\frac{y^2}{2\sigma_y^2}\right]\exp\left[-\frac{(z-H)^2}{2\sigma_z^2}\right]$$

$$C_{reflect}(x,y,-z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z u}\exp\left[-\frac{y^2}{2\sigma_y^2}\right]\exp\left[-\frac{(-z-H)^2}{2\sigma_z^2}\right]$$

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Ground surface reflection

$$C_{real}(x,y,z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z u}\exp\left[-\frac{y^2}{2\sigma_y^2}\right]\exp\left[-\frac{(z-H)^2}{2\sigma_z^2}\right]$$

$$C_{reflect}(x,y,-z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z u}\exp\left[-\frac{y^2}{2\sigma_y^2}\right]\exp\left[-\frac{(-z-H)^2}{2\sigma_z^2}\right]$$

$$C(x,y,z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z u}\exp\left[-\frac{y^2}{2\sigma_y^2}\right]\left\{\exp\left[-\frac{(z-H)^2}{2\sigma_z^2}\right]+\exp\left[-\frac{(z+H)^2}{2\sigma_z^2}\right]\right\}$$

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Mixing Height

- Air pollutants released at ground level will be mixed up to the mixing height, but not above it because of the extremely stable atmosphere above the mixing height.
- There is no upward flow above the mixing height.
- Can be determined experimentally or using empirical equations

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Typical mixing heights for the contiguous United States

Time	Mixing height (m)		
	Min	Max	Average
Summer morning	200	1100	450
Summer afternoon	600	4000	2100
Winter morning	200	900	470
Winter afternoon	600	1400	970

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$$z_{mix} = C_0 \frac{u_*}{2\Omega sin\phi} \quad \text{(Neutral atmosphere)}$$

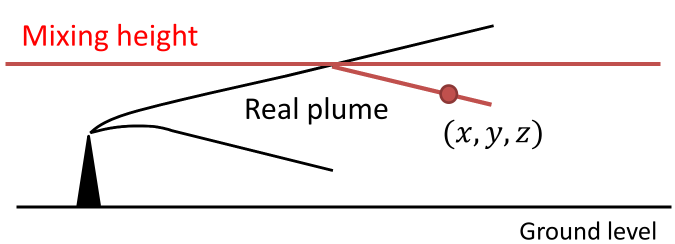
where C_0 is a coefficient that varies from 0.2 to 0.4;
u_* is the friction speed.
The term $(2\Omega \cdot \sin\phi)$ in the denominator stands for the Coriolis force because of the rotation of the earth. $\Omega = 7.27 \times 10^{-5}$ rad/s (Serway et al, 2000) is the angular speed of the earth and ϕ is the latitude where the air is of concern.

$$z_{mix} = C_s u_*^{1.5} \quad \text{(Stable atmosphere)}$$

$$C_s = 2400\ m^{0.5} s^{1.5}$$

$$z_{mix} = C_u \frac{u_*^{1.5}}{\sqrt{L(2\Omega sin\phi)^3}} \quad \text{(Unstable atmosphere)}$$

Mixing height reflection



$$C_{real}(x,y,z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z u} \exp\left[-\frac{y^2}{2\sigma_y^2}\right] \exp\left[-\frac{(z-H)^2}{2\sigma_z^2}\right]$$

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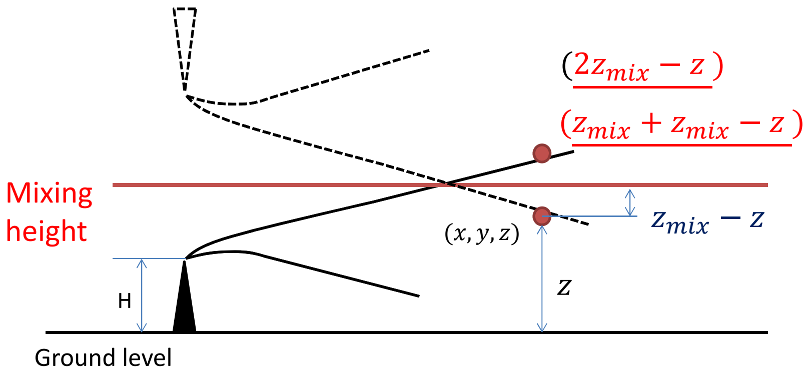
Mixing height reflection



$$C_{reflect}(x,y,-z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z u}\exp\left[-\frac{y^2}{2\sigma_y^2}\right]\underline{\exp\left[-\frac{(-(2z_{mix}-z)-H)^2}{2\sigma_z^2}\right]}$$

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Ground Surface AND Mixing Height Reflections

$$C(x,y,z) = \frac{\dot{m}}{2\pi\sigma_y\sigma_z u}\exp\left[-\frac{y^2}{2\sigma_y^2}\right]$$

$$\underline{\sum_{j=-\infty}^{j=\infty}\left\{\exp\left[-\frac{(z-H+2jz_{mix})^2}{2\sigma_z^2}\right]+\exp\left[-\frac{(z+H+jz_{mix})^2}{2\sigma_z^2}\right]\right\}}$$

- In practice $(j = -1, 0, 1)$ is sufficient for $\sigma_z < z_{mix}$
- $(j = -2, -1, 0, 1, 2)$ for $\sigma_z < 1.2z_{mix}$

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Complete vertical mixing between ground surface and mixing height

$$C(x,y) = \frac{\dot{m}}{\sqrt{2\pi}\sigma_y z_{mix} u} \exp\left[-\frac{y^2}{2\sigma_y^2}\right] \qquad \sigma_z > z_{mix}$$

- 1.5% error

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Gaussian Puff Model

Consider an instantaneous short-term release of air pollutant from a stack, where the mass of air pollutant released is $m(kg)$.

$$C = \frac{m}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z} \exp\left[-\frac{x^2}{2\sigma_x^2}\right] \exp\left[-\frac{y^2}{2\sigma_y^2}\right] \exp\left[-\frac{(z-H)^2}{2\sigma_z^2}\right]$$

- The Gaussian puff model is useful in safety analysis of accidental release of air pollutants and other chemicals rather than a continuous release of air pollutants. Readers are referred to literature for in-depth understanding of these topics.